Eur. Phys. J. A **14**, 349–353 (2002)
DOI 10.1140/epja/i2002-10006-0

THE EUROPEAN PHYSICAL JOURNAL A

Shell model interaction between proton holes and between proton holes and neutron particles around ^{208}Pb

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Received: 2 April 2002 / Revised version: 18 April 2002
Communicated by Th. Walcher

Abstract. An effective residual interaction between particles and holes for shell model calculations around ^{208}Pb, derived from the interaction between free nucleons, is compared with the measured properties of proton-hole neutron states in ^{208}Tl and the interaction between proton holes is adjusted to newly measured level energies in ^{206}Hg. These interaction elements are particularly relevant for neutron-rich nuclei. The adjustment of two mixing elements reproduces the known γ-decay data in ^{208}Tl.

PACS. 21.60.Cs Shell model – 21.30.Fe Forces in hadronic systems and effective interactions – 27.80.+w $190 \leq A \leq 219$

1 Introduction

Experiments to study the level schemes of neutron-rich or proton-deficient nuclei around ^{208}Pb have now become possible by various methods [1–6]. But so far spins and parities could not be assigned from experiment. Therefore one has already to rely on theory, which is here the shell model, to assign them. Also, a comparison between measured properties and model calculations is needed for a deeper understanding of the structure of these nuclei. The energies and wave functions from shell model calculations depend of course on the single-particle energies and the residual interaction between the particles. The aim of this article is to explore this interaction for the so far little studied part of the interaction between two proton holes and that between a neutron particle and a proton hole. The third component of importance for neutron-rich nuclei, namely, the neutron-neutron interaction, has already been treated [7].

Kuo and Brown [8] developed the procedure to calculate the interaction inside a finite nucleus from that between free nucleons, and Kuo and Herling [9] then calculated the matrix elements of this interaction for the region around ^{208}Pb. As the doubly magic core ^{208}Pb is not excited in the lowest states of the neighbouring nuclei, they limited their calculations to the interaction between particles in orbitals above the shell closure in^{208}Pb, and between holes, the orbitals below ^{208}Pb. This calculated interaction has been checked against experimental data, then improved and adjusted [7,10,11], and reproduces many experimental data well. It is used in the following for ^{206}Hg as given in ref. [11] in comparison with the recent experimental data from ref. [6].

An interaction between particles and holes has been calculated recently [12] from the H7B interaction [13], using the A-dependence of the parameters as given in this reference, and in the same way from the M3Y interaction [14]. Both interactions, which are based on that between free nucleons, give very similar results. The calculations are analogous to those by Kuo and Herling [9], but include only the bare matrix elements without any contributions from core polarization. The model space includes the orbitals between the $^{132}_{50}$Sn$_{82}$ core and $^{310}_{126}$X$_{184}$. The H7B interaction is used here for ^{208}Tl. The few experimental data for ^{208}Tl are from ref. [15].

In a rigorous theoretical treatment the single-particle energies and any operators would be determined together with the residual interaction. However here, the calculated interactions are to be combined and replaced, whenever possible, by experimental data in a consistent way. The levels of ^{206}Hg are treated as just two proton holes without any admixed excitations of the ^{208}Pb core and ^{208}Tl as one proton hole and one neutron only. Accordingly the single-particle energies are chosen to reproduce the energy of the appropriate state in the one-particle (hole) nucleus, if this is regarded as a pure state without admixed core excitations. Also the electromagnetic matrix elements give the measured properties of the single-particle states, if these are considered as being pure. For instance, the

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single-particle energy of the $g_{9/2}$ neutron gives directly the energy of the $9/2^+$ ground state of ^{209}Pb and its $M1$ matrix element the magnetic moment of the ^{209}Pb ground state. All energies of the single-particle states have been measured [16] and these values are used. The set of $M1$ and $E2$ matrix elements in the appendix of ref. [16] consists of measured and calculated values, that are in accordance with this treatment. The influence of the size of the configuration space is also considered below.

Besides the main goal, to test and establish a practical interaction, perhaps, some hints on improvements of the calculations of realistic interactions might be gained from this comparison with experiment. Realistic interactions are calculated at present with improved mathematical procedures from better free nucleon-nucleon potentials. The state of these calculations has been summarized by Covello *et al.* [17]. Around ^{208}Pb the interaction between like particles and that between like holes, isospin $T = 1$ only, has been calculated in this way. The results for proton particles [18] reproduce the experiment well. The shell structure of very neutron-rich nuclei is predicted to change appreciably [19]; a good knowledge of the "normal" structure is needed for comparison.

2 The experimental data

The interaction between neutron particles and proton holes is seen in ^{208}Tl. The six lowest levels and their γ-decay, as presented in ref. [15], are known from α- and γ-spectroscopy in the decay chain of ^{232}Th. These states are shown in fig. 1. They are all of positive parity and belong to the configurations $\nu g_{9/2}\pi s_{1/2}^{-1}$ and $\nu g_{9/2}\pi d_{3/2}^{-1}$. Nothing else is known on ^{208}Tl.

The 2^+ and 5^- levels in ^{206}Hg, a nucleus that is also very hard to reach experimentally, had been known [20]. Now three more pure two-hole levels 7^-, 8^+, and 10^+ have been found [6], and the level scheme along the yrast line has been extended to 13^-.

3 The interaction

3.1 Pandya transform and procedure

The low-lying states in ^{208}Tl consist of one neutron particle added to the ^{208}Pb core and one proton hole removed from it. Therefore, the experimental data are best interpreted in the particle-hole formalism. But any more general calculations of other nuclei in the vicinity or the two-particle–two-hole states in ^{208}Tl are more easily calculated with particle-particle interaction elements. Shell model codes, like OXBASH [21] use the particle-particle formalism. The Pandya transformation relates the matrix elements in both descriptions:

$$E_I(j_1 j_4^{-1}; j_3 j_2^{-1}) = -(-1)^{j_1+j_2+j_3+j_4} \sum_J (2J+1) W(j_1 j_2 j_4 j_3; JI) E_J(j_1 j_2; j_3 j_4),$$

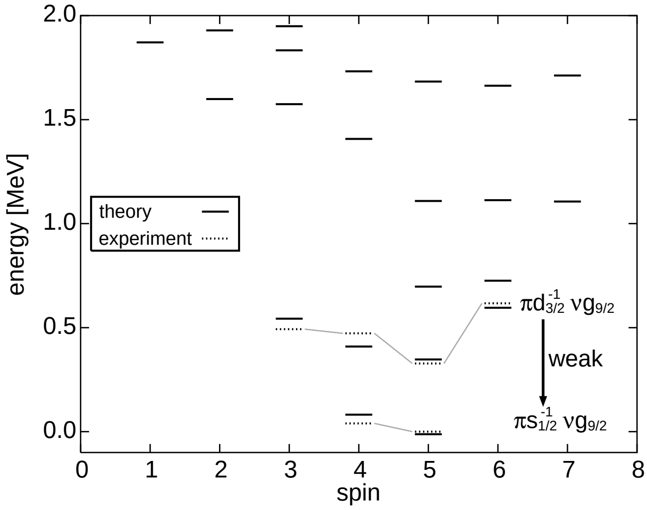

Fig. 1. Positive-parity states in ^{208}Tl. The energies, calculated with the original (unadjusted) interaction and the six known experimental states are shown.

where j_1 to j_4 stand for particles, j_4^{-1} means a hole in the orbital j_4. Because j_2 and j_4 are interchanged between holes and particles, the Pandya transformation relates states that one naively would not expect to be connected. For instance, the parity of the states can be different on the two sides of the equation.

The set of particle-particle matrix elements of the original calculated interaction [12], which corresponds to interactions in ^{208}Tl, has been converted to particle-hole elements. The Schroedinger equation for one-particle–one-hole states is then trivially constructed and can be solved easily. The eigenvalues are compared to the experimental energies of the states. The γ-transitions ($M1$ and $E2$) are then calculated from the wave functions and compared to the experiment. The set of single-particle $M1$ and $E2$ elements of ref. [16] is used to calculate the transition elements between the particle-hole states by angular momentum recoupling [16]. The matrix elements of the particle-hole interaction are then adjusted by trial and error, until agreement with experiment is achieved. Finally one can go back to particle-particle interactions with the inverse Pandya transformation. For ^{206}Hg, the interaction between two holes is simply identical to that between two particles.

3.2 Interaction between neutrons and proton holes, ^{208}Tl

The six positive-parity levels, known from experiment, are compared in fig. 1 with the positive-parity states, as calculated from the original interaction. The agreement is good. According to the calculations, the six known states belong to quite pure configurations $\pi s_{1/2}^{-1}\nu g_{9/2}$ and $\pi d_{3/2}^{-1}\nu g_{9/2}$. These two configurations account for more than 96% in all six states. The γ-decay proceeds with the weak, l-forbidden $\langle s_{1/2} \parallel M(M1) \parallel d_{3/2}\rangle$ matrix element

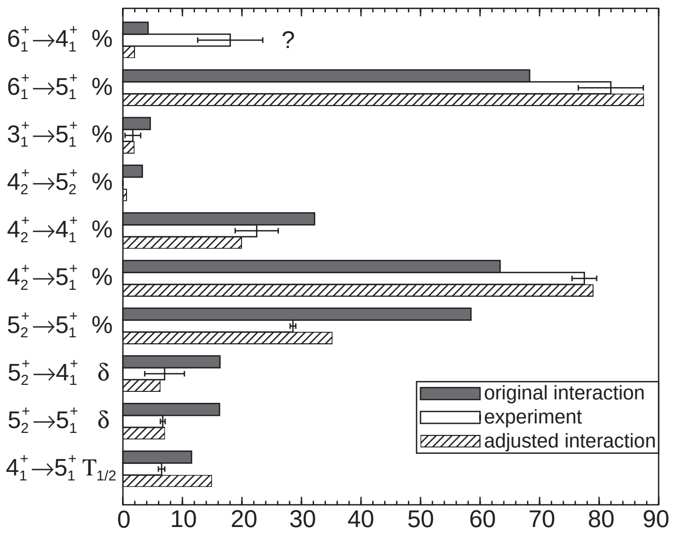

Fig. 2. All measured γ-decay properties in ^{208}Tl are compared with calculations with the original and adjusted interaction. The intensities of branches from a given state are marked by %, $M1/E2$ mixing ratios by $\delta = \delta(E2/M1) \times 100$, and the halflife in ps of the 4_1^+ level by $T_{1/2}$. The branch from a given level that adds to 100% is not presented.

or the equally weak spin flip $E2$ element between these orbitals. Configuration mixing, however, allows the transitions to proceed with strong diagonal elements. Configuration mixing occurs only for the 4^+ and 5^+ states, and therefore, all γ-transitions are completely determined by just two mixing ratios. The 5^+ and 6^+ levels contain small admixtures of $\pi s_{1/2}^{-1} \nu i_{11/2}$, but these contribute negligibly to the γ-transitions.

The mixing of the 4^+ and 5^+ states has been varied and the $M1$- and $E2$ γ-transitions calculated until good agreement with experiment was accomplished. After the mixing was determined, the diagonal matrix elements were adjusted to reproduce the measured level energies within a few keV. Figure 2 compares the measured γ-decay properties with those calculated for the original and the adjusted interaction. Taking into account the experimental errors, agreement is achieved for eight measured properties by modifying the two mixing elements. The transition from 6^+ to 4_1^+ is reported as doubtful in the experiment and is not well reproduced. The sign convention of the $M1/E2$ mixing ratio has not been checked; it is only significant that the signs for the two transitions are the same in experiment and calculation. The halflife of the 4_1^+ level has been measured by a rarely used method by Sevier [22]. The calculated value, using a total conversion coefficient $\alpha_{\text{tot}} = 24.4$ [23], does not agree. Indeed, the measured lifetime cannot be reproduced by any mixing. The $M1$ matrix elements of the $s_{1/2}$ proton and $g_{9/2}$ neutron that determine the transition for the dominant component are precisely measured. So there is a discrepancy between the basic understanding of the struture and of the experiment.

The 12^+ yrast state [6] in ^{206}Hg has mainly the stretched configuration $\pi(h_{11/2}d_{3/2})^{-1}\nu g_{9/2}p_{1/2}^{-1}$. Its calculated energy is too low. As all other interactions for this configuration are known from experiment, the repulsion in the 10^- state of the $\pi h_{11/2}^{-1}\nu g_{9/2}$ configuration has been increased by 145 keV. Due to the admixed configurations this value is not well determined. But this is one more example, that the repulsion in the highest-spin states is too small in the calculated interaction [24]; particularly in the analogous case of the $\pi i_{13/2}\nu i_{13/2}^{-1}$ 13^+ state in ^{208}Bi [25], where a very similar shift of +156 keV is needed too.

Table 1. Adjustment of the interaction in ^{208}Tl to reproduce the energies and γ-transitions of the lowest 6 states. Presented are: H7B/M3Y/adjusted matrix elements in keV.

$\langle \nu g_{9/2}\pi s_{1/2}; 4^+ \Vert H \Vert \nu g_{9/2}\pi s_{1/2}; 4^+ \rangle$	=	270/357/261
$\langle \nu g_{9/2}\pi s_{1/2}; 5^+ \Vert H \Vert \nu g_{9/2}\pi s_{1/2}; 5^+ \rangle$	=	142/161/197
$\langle \nu g_{9/2}\pi d_{3/2}; 3^+ \Vert H \Vert \nu g_{9/2}\pi d_{3/2}; 3^+ \rangle$	=	406/410/354
$\langle \nu g_{9/2}\pi d_{3/2}; 4^+ \Vert H \Vert \nu g_{9/2}\pi d_{3/2}; 4^+ \rangle$	=	206/207/239
$\langle \nu g_{9/2}\pi d_{3/2}; 5^+ \Vert H \Vert \nu g_{9/2}\pi d_{3/2}; 5^+ \rangle$	=	137/141/75
$\langle \nu g_{9/2}\pi d_{3/2}; 6^+ \Vert H \Vert \nu g_{9/2}\pi d_{3/2}; 6^+ \rangle$	=	432/425/454
$\langle \nu g_{9/2}\pi s_{1/2}; 4^+ \Vert H \Vert \nu g_{9/2}\pi d_{3/2}; 4^+ \rangle$	=	109/206/170
$\langle \nu g_{9/2}\pi s_{1/2}; 5^+ \Vert H \Vert \nu g_{9/2}\pi d_{3/2}; 5^+ \rangle$	=	$-46/-77/-120$

Six diagonal and two nondiagonal elements of the residual interaction have been determined in this way. They are presented in table 1 in comparison with those calculated from the H7B and M3Y interaction. Level energies and the γ-decays have been also calculated for only the main configurations $\nu g_{9/2}\pi s_{1/2}$ and $\nu g_{9/2}\pi d_{3/2}$, to see how sensitive the results depend on the configuration space. The offdiagonal elements and the diagonal elements for $\nu g_{9/2}\pi s_{1/2}$ are not affected, while the diagonal $\nu g_{9/2}\pi d_{3/2}$ elements would have to be lowered by 50 keV, except for 5^+ that is unchanged.

An adjusted interaction has been derived, substituting these directly determined matrix elements for the original, calculated elements. Otherwise all elements of the original interaction are left unchanged, as the average shift of the diagonal elements for positive parity is only 2 keV, and from two nondiagonal elements one cannot generalize. The calculated levels of both parities are presented in fig. 3, as it might help with future experiments. The 9^- level is predicted to be an isomer with $T_{1/2} \geq 10$ μs, which decays with a low-energy, highly converted, and configuration-hindered $M2$-transition. This makes spectroscopy along the yrast line difficult.

Kim and Rasmussen [26] calculated energies and wave functions for ^{208}Tl. They used a phenomenological Gaussian potential including a tensor force, that had been adjusted to ^{210}Bi and ^{210}Po. They find very similar configuration mixing for the 4^+ and 5^+ levels as we deduce from the γ-decay. Their eigenfunctions for the lowest states are in comparison with our values in brackets:

$$4^+ 0.93(0.93)\pi s_{1/2}^{-1}\nu g_{9/2} + 0.36(0.37)\pi d_{3/2}^{-1}\nu g_{9/2} + \dots,$$

$$5^+ 0.95(0.92)\pi s_{1/2}^{-1}\nu g_{9/2} - 0.30(0.39)\pi d_{3/2}^{-1}\nu g_{9/2} + \dots$$

The wave functions of the second states are orthogonal.

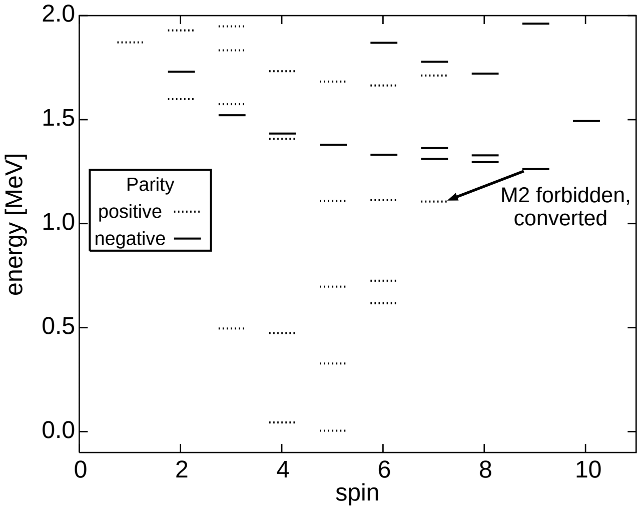

Fig. 3. Levels of ^{208}Tl calculated with the adjusted interaction, both positive and negative parity.

3.3 Interaction between proton holes, ^{206}Hg

Only the lowest-energy levels, 0^+ (ground state), 2^+, and 5^- had been known in ^{206}Hg [20]. Rydstroem *et al.* [11] tested the Kuo-Herling interaction between two proton holes against the energies of these levels. They adjusted the interaction to reproduce these energies by multiplying all 0^+ matrix elements by 0.876, and shifting the decisive diagonal elements for the 2^+ and 5^- states. We adjust the diagonal elements for the new 7^-, 8^+, and 10^+ levels to reproduce their energies in the same way. The corrections are:

$$177\,\text{keV} \quad \text{for} \quad \langle s_{1/2}d_{3/2}; 2^+\|H\|s_{1/2}d_{3/2}; 2^+\rangle\,,$$
$$15\,\text{keV} \quad \text{for} \quad \langle s_{1/2}h_{11/2}; 5^-\|H\|s_{1/2}h_{11/2}; 5^-\rangle\,,$$
$$87\,\text{keV} \quad \text{for} \quad \langle d_{3/2}h_{11/2}; 7^-\|H\|d_{3/2}h_{11/2}; 7^-\rangle\,,$$
$$5\,\text{keV} \quad \text{for} \quad \langle h_{11/2}h_{11/2}; 8^+\|H\|h_{11/2}h_{11/2}; 8^+\rangle\,,$$
$$68\,\text{keV} \quad \text{for} \quad \langle h_{11/2}h_{11/2}; 10^+\|H\|h_{11/2}h_{11/2}; 10^+\rangle\,.$$

The measured [27] transition strengths $B(E3, 5^- \rightarrow 2^+) = 0.18(2)$ W.u. and $B(E3, 10^+ \rightarrow 7^-) = 0.26(3)$ W.u. [6] provide information on nondiagonal elements. In the two-proton hole space the only possible $E3$-transition is between $h_{11/2}$ and $d_{5/2}$, and the $B(E3, \pi h_{11/2} \rightarrow d_{5/2}) = 25$ W.u. is inferred from the analogous $\nu j_{15/2} \rightarrow g_{9/2}$ transition and further systematics [28]. The uncertainty of this estimate is 20%. Because the 10^+ state is pure $h_{11/2}^{-2}$, the $B(E3, 10^+ \rightarrow 7^-)$ determines directly the admixture of $h_{11/2}d_{5/2}$ to the dominant component $h_{11/2}d_{3/2}$. The calculated amplitude of the admixture gives 0.4 W.u. within a factor 2 of the experiment.

The $5^- \rightarrow 2^+$ transition proceeds dominantly from the main component $h_{11/2}s_{1/2}$ of the 5^- state to the admixture of $d_{5/2}s_{1/2}$ in the 2^+ state. But $(h_{11/2}d_{3/2}) \rightarrow (d_{5/2}d_{3/2})$ interferes destructively. The calculated $B(E3, 5^- \rightarrow 2^+)$ is nearly 10 times the measured value. Therefore the amplitude of the $d_{5/2}s_{1/2}$ admixture in the 2^+ level should be decreased by a factor 0.4, (from -0.31 to -0.12), if one leaves the other admixtures constant. So there are 3 pieces of evidence, all suggesting that the mixing should be decreased, but by differing factors 0.86, 0.65 and 0.4.

We propose to adjust the interaction between proton holes from that of ref. [11] by using the now directly determined matrix elements. Furthermore all other diagonal matrix elements are adjusted by +70 keV, except for 0^+, because the 5 measured elements are all larger than the calculated ones with an average of 70 keV.

4 Conclusions

The neutron-rich, or proton-deficient, region around ^{208}Pb has been least studied so far, but becomes accessible to experiment now. An interaction between proton holes and between neutrons and proton holes has been checked against the few experimental data. It fits the data quite well. Some matrix elements could be directly determined from the measured properties of low-lying states in ^{208}Tl and ^{206}Hg, giving an improved interaction. These elements are also most important for the low-lying states in nuclei nearby. But some very neutron-rich nuclei like ^{204}Ir should be well described too, as the relevant orbitals are also $s_{1/2}$ and $d_{3/2}$ for protons and $g_{9/2}$ for neutrons. As the theoretical interaction reproduces the elements, that could be checked, reasonably, it should, in general, give meaningful results in guiding and interpreting new experimental results. One general adjustment is suggested, namely, raising all diagonal elements of the interaction between proton holes by +70 keV. One should also keep in mind that the states of highest spin with the particle and hole in high-spin orbitals are always calculated too low, independent of the type of particles. This has been shown here for a proton hole and neutron particle. The example of the 5^+ levels in ^{208}Tl shows that the configuration mixing is important. The original interaction reproduces the energies well, but only because deviations of the mixing and the diagonal elements compensate each other. A general discussion of the realistic interaction has to include neutron neutron-hole and proton proton-hole states in ^{208}Pb and proton neutron-hole levels in ^{208}Bi, for which more information is available.

K.H.M. thanks the colleagues at the Institue for Nuclear Physics, Krakow, Poland, where most of this work has been done, for their great hospitality and the Polish Science Foundation for supporting this stay. We thank B.A. Brown for providing the code and very valuable advice to calculate the realistic interaction and making the M3Y interaction available to us.

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